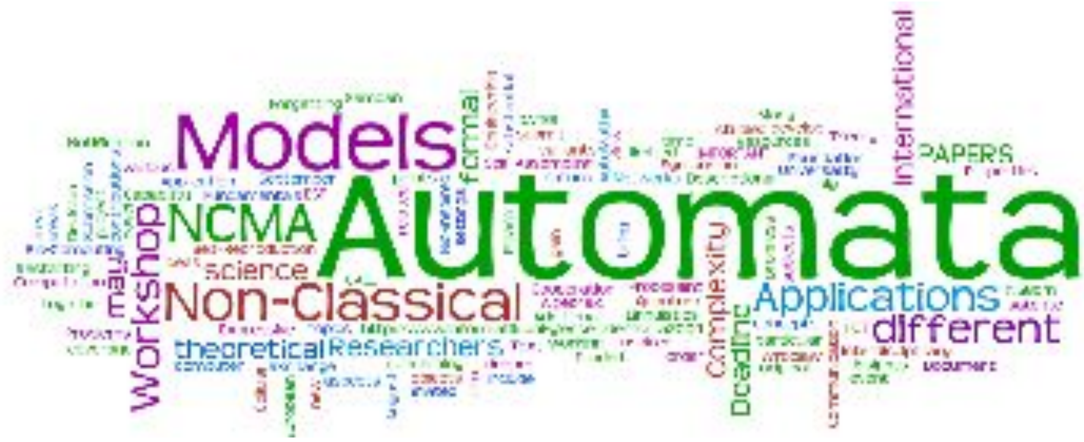

Recognition of uncountably many languages with one counter

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Our scope

Different bounded-error probabilistic models:
Error bound ε ($0 \leq \varepsilon < ½$):

- if $w \in L$, w is accepted with probability $1-\varepsilon$;
- if $w \notin L$, w is rejected with probability $1-\varepsilon$.

How many resources is enough for realtime PTMs and PCAs to recognize uncountably many languages?

$$\aleph_1$$

Realtime reading mode

Our realtime models operate in strict mode: any given input, say $w \in \Sigma^*$, is read as $\triangleright w \triangleleft$ from the left to the right and symbol by symbol without any pause on any symbol.

Uncountably many languages

- Logarithmic-space unary PTMs.
- Unary P2CAs.
- Unary PkCAs in $O(\sqrt[k-1]{n})$ space for k>2.
- loglog-space PTMs.
- Multicounter PCAs in $O(\sqrt[k]{\log n})$ space for any k≥1.
- P2CAs in $O(\sqrt[k]{n})$ space for any k>1.

Realtime PkCA

$$P = (S, \Sigma, \delta, s_1, s_a, s_r)$$

- S - the finite set of states,
- Σ - the input alphabet,
- δ: $S \times \Sigma \cup \{\triangleright, \triangleleft\} \times \{0,1\}^k \times S \times \{-1,0,1\}^k \to [0,1]$ - the transition function,
- $s_1 \in S$ - the initial state,
- $s_a \in S$ and $s_r \in S$ are the accepting and rejecting states, respectively.

Recognition of a language

Language $L \subseteq \Sigma^*$ is said to be recognized by a probabilistic machine P with error bound ε if:

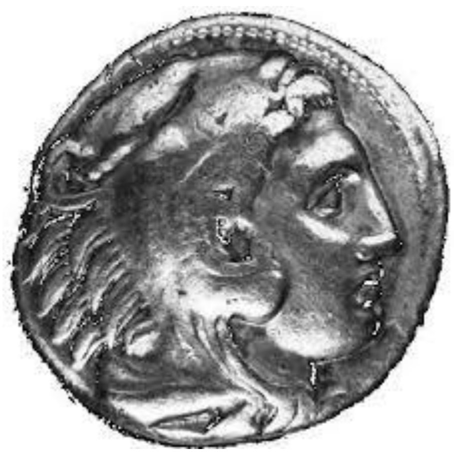
- any member is accepted by P with probability at least 1-ε,
- any non-member is rejected by P with probability at least 1-ε.

Space complexity

A language L is recognized by a bounded-error PkCA in space s(n), if the maximum absolute value of any of the counters is not more than s(n) for any input with length n.

Lemma for 64^k coin flips

- Let $x = x_1x_2x_3 \ldots$ be an infinite binary sequence. If a biased coin lands on head with probability $p = 0.x_101x_201x_301 \ldots$, then the value x_k can be determined with probability $\frac{3}{4}$ after 64^k coin tosses.



Lemma 2.0

Let $x = x_1\ x_2\ x_3\ \ldots$ be an infinite binary sequence. If a biased coin lands on head with binary probability value $p=0.\ x_1 01\ x_2 01\ x_3 01\ \ldots$, then the value x_k can be determined with probability at least $1 - 1/(4*2^l)$ after 64^k*2^l coin tosses, where $l>0$.

DIMA3$_l$(I)

$$\mathtt{DIMA3}_l = \{0^{2^0}10^{2^1}10^{2^2}1\cdots10^{2^{6k+l-2}}110^{2^{6k+l-1}}11^{2^{6k+l}}(0^{2^{3k+l}-1}1)^{2^{3k}} \mid k>0\}$$

$$\mathcal{I} = \{I \mid I \subseteq \mathbb{Z}^+\}$$

Let w_k be the k-th shortest member of DIMA3$_l$ for k>0.

$$\mathtt{DIMA3}_l(I) = \{w_k \mid k>0 \text{ and } k \in I\}$$

$DIMA3_l(I)$

$$w = 0^{t_1}10^{t_2}1\cdots10^{t_{m-1}}110^{t_m}11^{t'_0}0^{t'_1}10^{t'_2}1\cdots10^{t'_n}1$$

- 5 paths with equal probabilities.
- 4 paths check whether $w \in DIMA3_l$.
- 5th path calculates x_k. If $x_k=1$, accept the input with probability 5/9 and reject with probability 4/9; if $x_k=0$, reject the input.

$H = i*8^{k+1}*2^l + j*8^k*2^l + q = (8i+j)*8^k*2^l + q$, where $i\geq 0$, $j\in\{0,1,...,7\}$, and $q<8^k*2^l$.

$DIMA3_l(l)$

- If $w \in DIMA3_l(l)$, the input is accepted with probability at least $5/9 - 1/(36 * 2^l)$.
- If $w \notin DIMA3_l$, the input is rejected with probability at least $5/9$.
- If $w \in DIMA3_l$ and $w \notin DIMA3_l(l)$, the input is rejected with probability at least $5/9 - 1/(20 * 2^l)$.

Conclusion

For the recognition of uncountably many languages with bounded error realtime models:

- In case of unary alphabets we obtained positive result for P2CAs, while P1CAs can recognize only regular languages.
- In case of binary alphabets we obtained positive result for P1CAs, while PFAs can recognize only regular languages.

Thank you for your attention!
Ďakujem!